A State Estimation of the Standpipe of a Circulating Fluidized Bed using an Extended Kalman Filter

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1 Introduction

An Extended Kalman Filter(EKF) is used to estimate the state of the standpipe for a circulating fluidized bed(CFB). The dynamic model of the flow within the standpipe is based on mass conservation and a modified Richardson-Zaki correlation. The truncated Ergun equation is used to relate the pressure drop measurements to the amount and velocity of solids in the standpipe. The state estimation problem for nonlinear systems in the presence of process and measurement noise has been widely considered in the literatures and in applications. One of the most applied solutions is the Extended Kalman Filter(EKF), which consists of a Kalman filter obtained by a step-by-step linearization around the current estimate of the state vector. This research applies an extended Kalman filter as an estimator for the state of the standpipe for a circulating fluidized bed.

2 Mathematical Model

The model is based on conservation of mass as expressed by the one-dimensional continuity equation for both phases. If gas compressibility can be neglected the equations for both phases have the same form.

$$\frac{\partial \varepsilon_p}{\partial t} + \frac{\partial j_p}{\partial z} = 0\,,$$

where $p = g$ for gas or s for granular solid and ε is volume fraction and j is volumetric flux. If the two continuity equations are added and use is made of $\varepsilon_g + \varepsilon_s = 1$ we obtain the constraint that

$$j_0(t) = j_g(z,t) + j_s(z,t).$$

Usually j_0 is computed at a specified point, the inlet or exit, and is constant throughout the standpipe. The constraint reduces the model from two partial differential equations to one partial differential equation and an algebraic equation. When the standpipe is divided into cells, a discrete form of the controlling dynamic equation is obtained by integrating the gas phase equation over the i^{th} and replacing the time derivation by a forward difference. The resulting equation is:

$$\varepsilon_k^i = \varepsilon_{k-1}^i + \frac{\Delta t}{\Delta z}\left(j_{k-1}^{i-\frac{1}{2}} - j_{k-1}^{i+\frac{1}{2}}\right), \tag{1}$$

where ε_k^i is the void fraction at the center of the i^{th} cell at the time step k and $j_{k-1}^{i-\frac{1}{2}}$ is the volumetric gas flux between cell $i-1$ and cell i at the previous time step. The time increment and the length of each cell along the standpipe are Δt and Δz, respectively. The fluxes into and out of a cell are related to the void fraction ε in that cell by the upwind procedure. The relationship between ε_g and j_g is obtained by writing

$$\frac{j_g}{\varepsilon_g} - \frac{j_s}{\varepsilon_s} = V_r = V_t\,\zeta(\varepsilon),$$

and using an empirical expression that relates V_r to ε_g.

We define, $g(\varepsilon)$, $g'(\varepsilon)$ and $\zeta(\varepsilon)$ that are used in the calculation of the fluxes by

$$\begin{aligned}
g(\varepsilon) &= \varepsilon\,(1-\varepsilon)\,\zeta(\varepsilon) \\
g'(\varepsilon) &= \frac{\mathrm{d}g(\varepsilon)}{\mathrm{d}\varepsilon}, \text{ and} \\
\zeta(\varepsilon) &= \begin{cases} 0 & \text{if} \quad \varepsilon \le \varepsilon_{pb} \\ \frac{(\varepsilon_{mf})^{n-1}}{\varepsilon_{mf}-\varepsilon_{pb}}\,(\varepsilon - \varepsilon_{pb}) & \text{if} \quad \varepsilon_{pb} < \varepsilon < \varepsilon_{mf} \\ \varepsilon^{n-1} & \text{if} \quad \varepsilon \ge \varepsilon_{mf} \end{cases}.
\end{aligned}$$

ε_{mf} is the void fraction at minimum fluidization, and ε_{pb} is the void fraction in a packed bed. We further define the void fractions at a cell boundary to be the arithmetic mean of the void fractions in adjacent cells,

$$\bar{\varepsilon}_{k-1}^{i-\frac{1}{2}} = \frac{\varepsilon_{k-1}^{i-1} + \varepsilon_{k-1}^{i}}{2}.$$

The wave speed $\lambda^{i-\frac{1}{2}}$ of the void fraction disturbance at the boundary between cell $i-1$ and cell i is given by

$$\lambda^{i-\frac{1}{2}} = j_0 + V_t g' \left(\varepsilon_{k-1}^{i-\frac{1}{2}} \right) ,$$

where V_t is the terminal velocity of a single isolated particle. The wave can propagate either up or down in the standpipe:

$$\begin{aligned}
j_{k-1}^{i-\frac{1}{2}} &= j_0 \, \varepsilon_{k-1}^{i-1} + V_t \, g \left(\varepsilon_{k-1}^{i-1} \right) \quad \text{if} \quad \lambda^{i-\frac{1}{2}} > 0 \, , \\
j_{k-1}^{i-\frac{1}{2}} &= j_0 \, \varepsilon_{k-1}^{i} + V_t \, g \left(\varepsilon_{k-1}^{i} \right) \quad \text{if} \quad \lambda^{i-\frac{1}{2}} < 0 \, .
\end{aligned}$$

The measurement model that relates the pressure difference between cell i and the topmost cell of the standpipe at the time-step k is the numerically integrated version of the truncated Ergun equation. If we let

$$q(\varepsilon) = \frac{(\varepsilon - 1)^2}{\varepsilon^2} \zeta(\varepsilon) \, ,$$

then we can write the pressure difference p_k^i as

$$p_k^i = \frac{1}{2} C_1 V_t \sum_{\ell=i}^{N-1} \left(z_{\ell+1} - z_\ell \right) \left\{ q\left(\varepsilon_k^\ell \right) + q\left(\varepsilon_k^{\ell+1} \right) \right\} \, , \tag{2}$$

where N is the number of cells in the standpipe, C_1 is a constant $\frac{150\mu}{{d_{vs}}^2}$, μ is the fluid viscosity, and d_{vs} is the effective particle diameter.

3 An Extended Kalman Filter

The behavior of the standpipe in a CFB and its pressure measurements constitute a non-linear discrete-time system represented by two sets of equations:

$$\begin{aligned}
\boldsymbol{\varepsilon}_k &= \boldsymbol{f}_{k-1} \left(\boldsymbol{\varepsilon}_{k-1} \right) + \boldsymbol{w}_{k-1} \, , \\
\boldsymbol{p}_k &= \boldsymbol{h}_k \left(\boldsymbol{\varepsilon}_k \right) + \boldsymbol{v}_k \, .
\end{aligned}$$

The vector $\boldsymbol{f}(\cdot)$ is a non-linear function of the state in equation (1), and the vector $\boldsymbol{h}(\cdot)$ captures every term on the right side of equation (2). The noise terms, $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ are added to account for errors in the dynamic and measurement models, respectively. They are uncorrelated, $\mathrm{E}\left\{ \boldsymbol{w}_k \, {\boldsymbol{v}_k}^{\mathrm{T}} \right\} = 0$ and are assumed normally distributed with zero mean, $\mathrm{E}\left\{ \boldsymbol{w}_k \right\} = \mathrm{E}\left\{ \boldsymbol{v}_k \right\} = 0$ and with covariance matrices, $\mathsf{Q} = \mathrm{E}\left\{ \boldsymbol{w}_k \, {\boldsymbol{w}_k}^{\mathrm{T}} \right\}$, and $\mathsf{R} = \mathrm{E}\left\{ \boldsymbol{v}_k \, {\boldsymbol{v}_k}^{\mathrm{T}} \right\}$, respectively. For this research Q and R are diagonal positive semi-definite matrices tuned to provide good performance.

We define the *a priori* and the *a posteriori* estimate errors as $\boldsymbol{e}_{k(-)} = \boldsymbol{\varepsilon}_k - \hat{\boldsymbol{\varepsilon}}_{k(-)}$, and $\boldsymbol{e}_{k(+)} = \boldsymbol{\varepsilon}_k - \hat{\boldsymbol{\varepsilon}}_{k(+)}$, respectively. Then we define the *a priori* and the *a posteriori* estimate error covariance matrices as

$$\mathsf{P}_{k(-)} = \mathrm{E}\left\{\boldsymbol{e}_{k(-)}\,\boldsymbol{e}_{k(-)}{}^{\mathrm{T}}\right\}, \quad \text{and} \quad \mathsf{P}_{k(+)} = \mathrm{E}\left\{\boldsymbol{e}_{k(+)}\,\boldsymbol{e}_{k(+)}{}^{\mathrm{T}}\right\}, \quad \text{respectively.}$$

Because $\boldsymbol{\varepsilon}_k$ is never known, the EKF algorithm calculates $\mathsf{P}_{k(-)}$ and $\mathsf{P}_{k(+)}$ other ways, as will be shown.

4 EKF Algorithm

In overview, the Kalman filter takes into account information from both the dynamic model and the measurement model and performs a running, least-squares, error minimization to obtain the best estimate of the void fraction distributions. In general, the EKF uses the dynamic model to predict a set of $\boldsymbol{\varepsilon}_{k(-)}$ for a given time. It then uses the measurement model to correct that estimate to $\boldsymbol{\varepsilon}_{k(+)}$, the EKF estimate. In detail, the EKF algorithm is:

1. Assign values for the diagonal and positive semi-definite matrices Q and R, and choose initial conditions for $\hat{\boldsymbol{\varepsilon}}_{k(+)}$ and $\mathsf{P}_{k(+)}$.

2. The predicted state vector is determined from the dynamic model, Eq(1).

$$\hat{\boldsymbol{\varepsilon}}_{k(-)} = \boldsymbol{f}_{k-1}\left(\hat{\boldsymbol{\varepsilon}}_{k-1(+)}\right) \; .$$

3. The Jacobian matrix of the non-linear dynamic model is calculated as

$$\mathsf{F}_k\left(\hat{\boldsymbol{\varepsilon}}_{k(+)}\right) = \left.\frac{\partial \boldsymbol{f}_k(\boldsymbol{\varepsilon})}{\partial \boldsymbol{\varepsilon}}\right|_{\boldsymbol{\varepsilon}=\hat{\boldsymbol{\varepsilon}}_{k(+)}} \; .$$

4. The *a priori* error covariance matrix is computed as

$$\mathsf{P}_{k(-)} = \mathsf{F}_{k-1}\left(\hat{\boldsymbol{\varepsilon}}_{k-1(+)}\right)\mathsf{P}_{k-1(+)}\mathsf{F}_{k-1}\left(\hat{\boldsymbol{\varepsilon}}_{k-1(+)}\right)^{\mathrm{T}} + \mathsf{Q} \; .$$

5. The Jacobian matrix of the non-linear measurement model is calculated as

$$\mathsf{H}_k\left(\hat{\boldsymbol{\varepsilon}}_{k(-)}\right) = \left.\frac{\partial \boldsymbol{h}_k(\boldsymbol{\varepsilon})}{\partial \boldsymbol{\varepsilon}}\right|_{\boldsymbol{\varepsilon}=\hat{\boldsymbol{\varepsilon}}_{k(-)}} \; .$$

6. The Kalman gain matrix, K_k which minimizes the a posteriori error covariance, is computed as

$$\mathsf{K}_k = \mathsf{P}_{k(-)}\mathsf{H}_k\left(\hat{\boldsymbol{\varepsilon}}_{k(-)}\right)^{\mathrm{T}}\left\{\mathsf{H}_k\left(\hat{\boldsymbol{\varepsilon}}_{k(-)}\right)\mathsf{P}_{k(-)}\mathsf{H}_k\left(\hat{\boldsymbol{\varepsilon}}_{k(-)}\right)^{\mathrm{T}} + \mathsf{R}\right\}^{-1} .$$

7. The estimate of the pressure difference is determined from the measurement model, Eq.(2),

$$\hat{\boldsymbol{p}}_k = \boldsymbol{h}_k\left(\hat{\boldsymbol{\varepsilon}}_{k(-)}\right) .$$

8. The corrected estimate of the state vector is determined as

$$\hat{\boldsymbol{\varepsilon}}_{k(+)} = \hat{\boldsymbol{\varepsilon}}_{k(-)} + \mathsf{K}_k\left\{\boldsymbol{p}_k - \hat{\boldsymbol{p}}_k\right\} .$$

   This is the EKF estimate of the void fraction profile at time step k.

9. The *a posteriori* error covariance matrix is computed as

$$\mathsf{P}_{k(+)} = \left\{\mathrm{I}_N - \mathsf{K}_k\mathsf{H}_k\left(\hat{\boldsymbol{\varepsilon}}_{k(-)}\right)\right\}\mathsf{P}_{k(-)} ,$$

   where I_N is an $N \times N$ identity matrix.

10. Finally, the time step is incremented and we return to step 2.

5 Apparatus and Operating Condition

The circulating fluidized bed system that we are studying consists of a standpipe, a non-mechanical valve, a riser, and a gas/solid separator. The standpipe serves as a reservoir for solids as well as providing for pressure balance around the loop. It is made from clear acrylic so that the bed height can be observed and recorded. It is 37.2 ft long and 1 ft in diameter. It has eight pressure taps located at 1, 4, 7, 10, 11.5, 16, and 20.5 ft from the bottom. The solid used in this experiment is coke breeze of 200 μ average diameter with bulk density 54 lb/ft^3. It circulates at an average rate of 24×10^3 lbs/hr as measured by the turn rate of a short spiral device located near the bottom of the standpipe. The data from the pressure taps and the mass circulation rate are sampled at a 1 Hz rate by a data acquisition system that stores the data for off-line processing. Air injected at the base of the standpipe at 265 SCFH serves to drive the solids from the standpipe, through the non-mechanical valve, and into the base of the riser. This injected air is referred to as move air and is one of the primary controls of this system. In order to maintain pressure balance around the CFB, the move air splits with part of it passing through the non-mechanical valve and part flowing upward through the standpipe. Air injected at a rate of 70×10^3 SCFH at the base of the riser causes the solids entering the riser to be transported rapidly upward. The outflow from the top of the riser then passes to the gas/solid separator where the air exits the system and solids returned to the top of the standpipe where they fall onto the top of a dense bed of varying height. From there solids move downward as a dense bed. For the first 250 seconds the bed level in the standpipe is held at 11.3 ft. After that solids are fed into the riser which causes the bed level in the standpipe to rise gradually and obtain approximately 22 ft at 1100 seconds.

6 Standpipe State Estimation

The length of the standpipe is discretized into 25 cells each of length, Δz_i. The 24 cells from the bottom are 1.5 ft in length, and very top cell, the 25^{th} cell, is 1.2 ft long. The time increment is chosen to provide stable performance, $\Delta t = 0.1$. The theoretical value of V_t is calculated from particle and fluid properties, but in the computer simulations we find that half of this theoretical V_t, that is 1.92 ft/sec, gives better results. The value of n is calculated according to the standard Richardson-Zaki procedure and found to be 3.35. The parameters ε_{mf} and ε_{pb} are chosen to be 0.4 and 0.45, respectively. Since the simple model used in this paper does not predict the split of the move air, it is chosen to obtain good performance. The results shown here assume half of the move air goes up the standpipe. After a trial and error using computer simulation, we assign error covariance matrices to be $\mathsf{Q} = 0.0001\,\mathrm{I}_{25}$ and $\mathsf{R} = 10\,\mathrm{I}_7$. We choose the initial estimated void fraction to be $\hat{\varepsilon}_k^i(+) = 0.7$, $i = 1 \cdots 25$ and the initial *a posteriori* error covariance matrix to be $\mathsf{P}_{0(+)} = \mathrm{I}_{25}$. The EKF algorithm given in section 4 is implemented in MATLAB® code and run on a 450MHz Pentium® III computer, and it approximately takes 2400 seconds for a 1100 second simulation.

Figure 1 shows the initial condition and illustrates that the EKF finds the void fraction profile within the standpipe in less than 40 seconds. $\Diamond$ is initial conditions, $+$ is EKF estimate at 10 seconds, and $\bullet$ is EKF estimate at 40 seconds. The EKF estimates the bed height to be about 12 ft, compared with the observed value of 11.3 ft.

Figure 2 compares the estimated pressure profile($\bullet$) to the measured pressure profile($\times$). From low to high pressures, the first three measured values form a steep straight line indicating those pressures were measured above the bed level. The remaining four values are within the bed and form another straight line except for the pressure at 7 ft. We believe it is anomalously low because of the spiral solids flow measuring device. Both of the actual and the estimate agree on the bed level at approximately 12 ft.

Figure 3 shows the EKF estimate of the void fraction profile at 1100 seconds. After 250

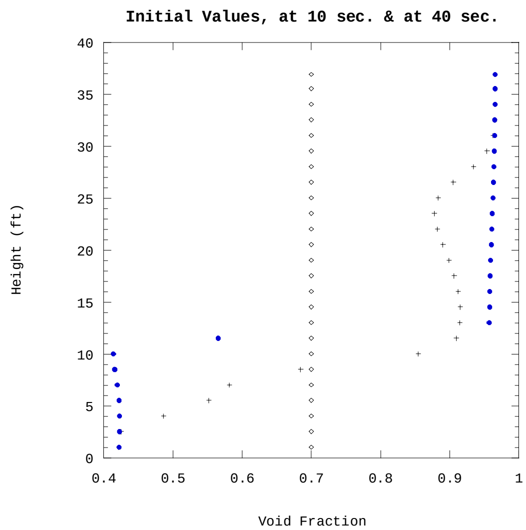

Figure 1: Estimated void fraction profiles when time is 0, 10, and 40 seconds.

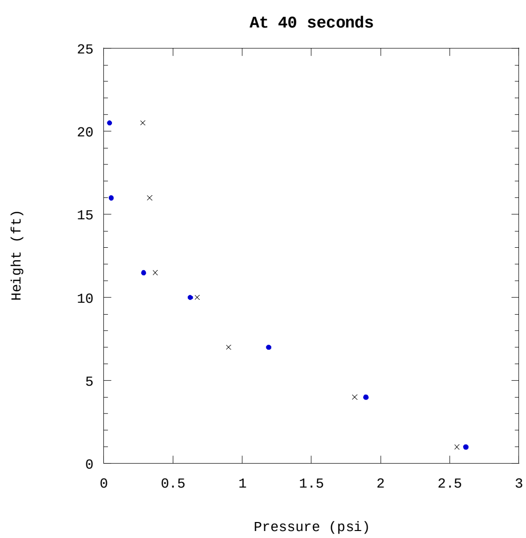

Figure 2: Measured($\times$) and estimated($\bullet$) pressure profiles at 40 seconds.

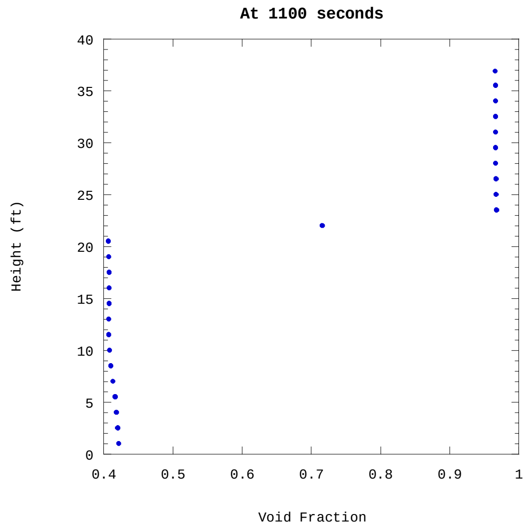

Figure 3: Void fraction profile at 1100 seconds.

seconds the standpipe begins to fill with solids. The EKF is found to correctly track the bed height in the standpipe while filling. The profile correctly reflects the observed bed height of 22 ft.

Figure 4 shows that all of the pressure taps are within the bed at 1100 seconds. After the initial 40 seconds, the EKF closely estimates the pressure profile in the standpipe. The biggest discrepancy between measured and estimate pressure is at 1 ft. We believe that this occurs because the EKF does not consider an obstacle in the bed that impedes the solid flow.

Figure 5. shows the solids inventory in the standpipe as calculated by $\sum_{i=1}^{25} \left(1 - \hat{\varepsilon}_{k}^{i}(+)\right) \Delta z_i$ for each time step k. The initial rapid change in the amount of solids is caused by the poor initial condition, $\hat{\varepsilon}_{k}^{i}(+) = 0.7$ for all i. The EKF needs approximately 40 seconds to find a good estimate of the amount of solids. Between 50 and 250 seconds, the estimated amount of solids is constant because the actual amount of solids in the system is constant as indicated by the constant bed level. While filling, the EKF responds by increasing its estimate of the amount of solids after a 40–50 second delay.

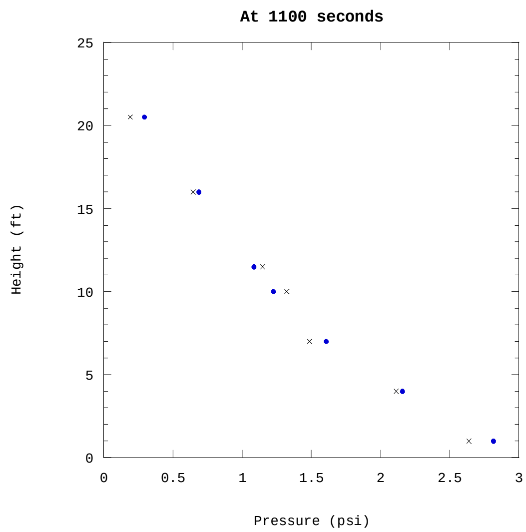

Figure 4: Measured($\times$) and estimated($\bullet$) pressure profiles at 1100 seconds.

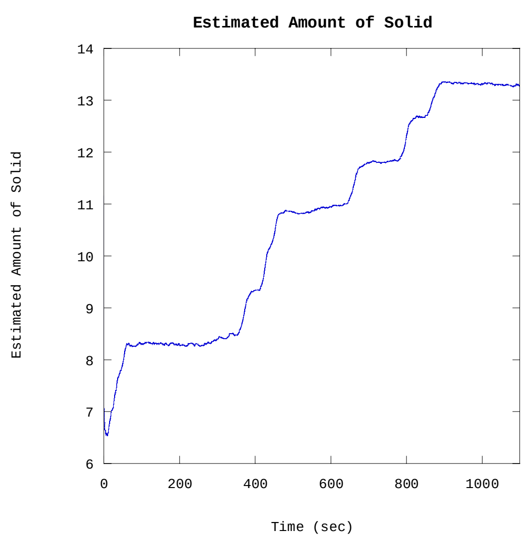

Figure 5: Estimated amount of solids in the standpipe.

7 Conclusion and Future Work

In this paper, the extended Kalman filter is successfully applied to estimate the void fraction and the pressure profiles in the standpipe for a circulating fluidized bed. We choose the values of solid circulation rate and move air split. Future work includes treating such things as the split of move air and solid flux as parameters to be estimated by the EKF, allowing multiple aeration ports, increasing the speed of the EKF algorithm, implementing it in real time, extending it to the entire CFB, and using it as a part of an optimal control scheme for the CFB.

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